 

Selected Paper

# Aromatic Conjugation Pathways in Porphyrins

**Jun-ichi Aihara,*[1] Eiichi Kimura,[1] and Tadeusz Marek Krygowski[2]**

[1]Department of Chemistry, Faculty of Science, Shizuoka University, Oya, Shizuoka 422-8529

[2]Department of Chemistry, University of Warsaw, ul. Pasteura 1, Pl-02 093 Warsaw, Poland

Received October 29, 2007; E-mail: scjaiha@yahoo.co.jp

The $\pi$-current density induced in a polycyclic $\pi$-system is strongly dependent on molecular geometry, so that information on main aromatic pathways cannot be extracted straightforwardly from the $\pi$-current density. Using our graph theory of aromaticity and ring-current diamagnetism, we re-interpreted the $\pi$-current densities and aromatic stabilization energies of porphyrins consistently and found that main pathways of $\pi$-electron circulation along the macroscopic ring are not those of aromatic stabilization. Four five-site circuits instead proved to be the main origin of aromaticity in porphyrins. In general, currents are induced in all possible circuits in a $\pi$-system. Superposition of all these circuit currents gives rise to the apparent bifurcation of the $\pi$-current across each pyrrolic unit. Local $\pi$-currents induced in pyrrolic rings represent those induced in the corresponding five-site circuits.

Electronic and magnetic properties of porphyrins have so far been investigated extensively, because they have many important biological functions.[1,2] The parent compounds of porphyrins are free-base porphine (**1**) and metalloporphine **2**, the structural formulae of which are presented in Figure 1. We here confine ourselves to the study of their aromaticity and ring-current diamagnetism. Several conjugation pathways have been proposed for the flow of $\pi$-electrons through the 24-site-conjugated $\pi$-system of the porphine macrocycle.[3–7] Traditionally, free-base porphyrins have been described as bridged diaza[18]annulenes with two localized $\beta$–$\beta'$ pyrrolic double bonds.[6,7] Metalloporphyrins have sometimes been represented as dianions that contain a 16-atom inner conjugation pathway (i.e., internal cross).[3–5] These conjugation or delocalization pictures emphasize that $18\pi$ conjugation pathways contribute predominantly to the aromaticity of porphyrins. Typical aromatic conjugation pathways so far proposed for **1** and **2** are summarized in Figure 2, in which **1a** and **2a** represent $18\pi$ conjugation pathways.

Many chemists have discussed conjugation pathways in porphyrins in structural terms and with respect to ring-current effects.[3–13] At the turn of the century (1998–2002), three research groups re-investigated possible aromatic pathways for porphyrins by carrying out sophisticated ring-current and nucleus-independent chemical shift (NICS) calculations.[8–13] They all noted that not only conjugation along the porphyrin macrocycle but also local aromaticity in pyrrolic rings is important in determining the degree of global aromaticity in porphyrins. Such a viewpoint is graphically shown as **1b** and **2b** in Figure 2. The aim of this work is to explore the main aromatic

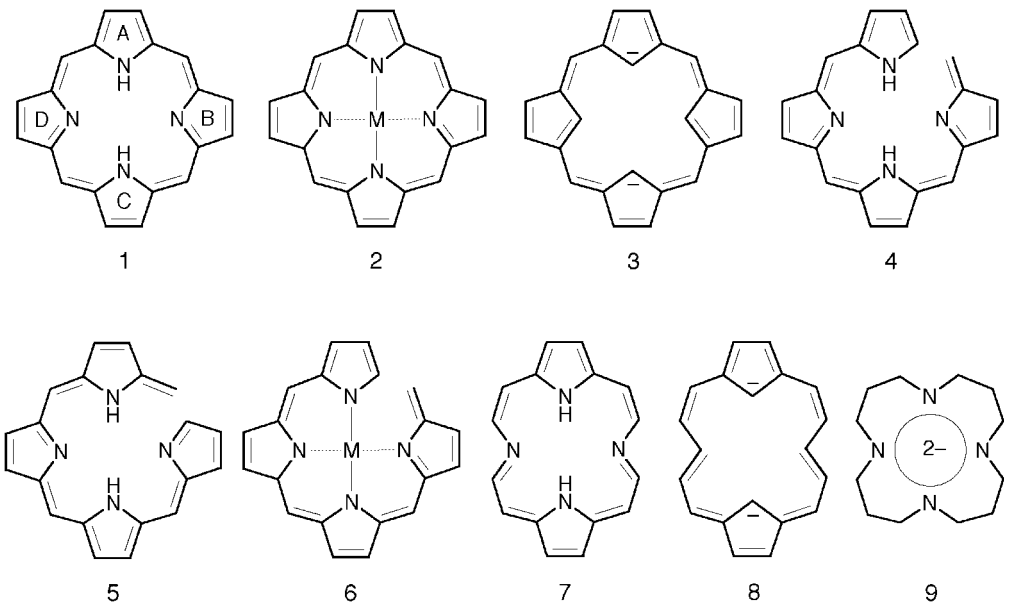


**Figure 1.** Porphyrins and related species studied.

Published on the web July 10, 2008; doi:10.1246/bcsj.81.826

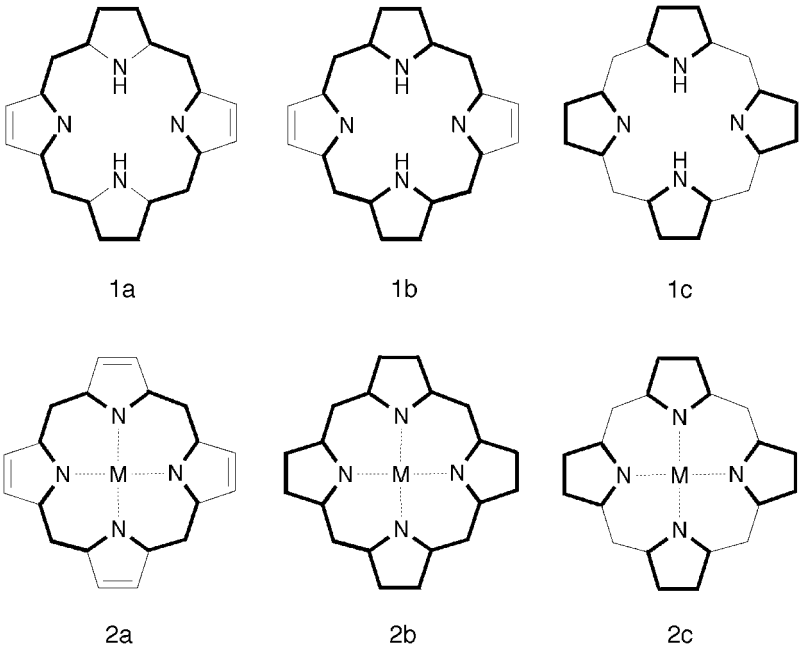


**Figure 2.** Main conjugation pathways proposed for free-base porphine **1a–1c** and metalloporphine **2a–2c**.

conjugation pathways of porphyrins by applying our graph theory of aromaticity[14–21] and ring-current diamagnetism[22–34] to these molecules. In particular, the classical 18$\pi$-[18]annulene and 18$\pi$-[16]annulene pathways in porphyrins will be examined critically in graph-theoretical terms.

## Theory

We first survey our theory of ring-current diamagnetism needed to calculate ring-current diamagnetic susceptibilities and related quantities. It is a graph-theoretical variant[22–34] of Hückel–London theory.[35,36] We assume that the central metal ion (e.g., the magnesium ion) in **2** is not included in the $\pi$-system and that all nitrogen atoms coordinated to the metal ion are of imine (=N–) type. Van-Catledge's set of Hückel parameters for heteroatoms[37] is used.

**Ring-Current Diamagnetic Susceptibility.** Our graph theory allows a partitioning of the ring-current diamagnetic susceptibility for a polycyclic $\pi$-system exactly into circuit contributions. This has interpretive advantages in many cases. Here, circuits stand for all possible cyclic or closed paths that can be chosen from a cyclic $\pi$-system.[38] For example, two six-site circuits and one ten-site one can be chosen from the naphthalene $\pi$-system.

We first evaluate the value of $A_i$ defined for each circuit in a heterocyclic $\pi$-system as:[23,29,30]

$$A_i = 4\prod_{m>n}^{r_i} k_{mn} \sum_{j}^{occ} \frac{P_{G\text{-}r_i}(X_j)}{P'_G(X_j)} \tag{1}$$

Here, $r_i$ is a set of conjugated atoms and $\pi$-bonds that constitute the $i$-th circuit $c_i$; $k_{mn}$ is the Hückel parameter for the resonance integral between atoms $m$ and $n$; $m$ and $n$ run over all $\pi$-bonds that belong to $r_i$; G-$r_i$ is the subsystem of G, obtained by deleting $r_i$ from G; $P_G(X)$ and $P_{G\text{-}r_i}(X)$ are the characteristic polynomials for G and G-$r_i$, respectively; $X_j$ is the $j$-th largest zero of $P_G(X)$; a prime added to $P_G(X)$ indicates the first derivative with respect to $X$; and $j$ runs over all occupied $\pi$ molecular orbitals. If there are degenerate $\pi$ molecular orbitals, eq 1 must be replaced by others.[23,24,27,28]

Ring-current diamagnetic susceptibility ($\chi_G$) is defined as a second derivative of total $\pi$-electron energy with respect to the intensity of the applied magnetic field. When an external magnetic field, $H$, is oriented perpendicular to the plane of G, the ring-current susceptibility can be expressed in the form:[23–29]

$$\chi_G = 4.5\chi_0 \sum_{i}^{G} A_i \left(\frac{S_i}{S_0}\right)^2 \tag{2}$$

where $\chi_0$ is the ring-current susceptibility of benzene; $S_i$ and $S_0$ are the areas of $r_i$ and the benzene ring, respectively. Positive and negative $A_i$ values represent diamagnetic and paramagnetic contributions, respectively. $\chi_G$ agrees exactly with the value calculated using conventional Hückel–London theory.[35,36] Thus, the ring-current susceptibility can be evaluated additively with respect to individual circuits. The contribution of the $i$-th circuit to $\chi_G$ i.e., the circuit-current susceptibility is then given as[23–28]

$$\chi_i = 4.5\chi_0 A_i \left(\frac{S_i}{S_0}\right)^2 \tag{3}$$

**Circuit Current.** A current induced in each circuit may be termed a circuit current. According to electromagnetism, magnetization, $M$, due to a loop current, $I$, induced by a magnetic field, $H$, is given by $IS$, where $S$ is the area enclosed by the loop.[39] Since $M = \chi H$, $I$ must be equal formally to $\chi H/S$, the $i$-th circuit-current susceptibility, $\chi_i$, must correspond to the induction of a $\pi$-electron current in the $i$-th circuit the intensity of which is given by:[28–30]

$$I_i = 4.5 I_0 A_i \frac{S_i}{S_0} \tag{4}$$

where $I_0$ is the intensity of a $\pi$-electron current induced in the

benzene ring. Positive and negative $A_i$ values indicate diatropic and paratropic currents, respectively. A current density map for an entire $\pi$-system is obtained by superposing all the circuit currents.

**Circuit Resonance Energy.** Bosanac and Gutman defined the cyclic conjugation energy ($CCE_i$) arising from the $i$-th circuit as the difference in total $\pi$-electron energy between G and G-$c_i$.[40–43] This energy difference is attributable to the energy gain or loss due to cyclic conjugation along the $i$-th circuit. For polycyclic aromatic species, our $A_i$ value is comparable in magnitude to $CCE_i$.[32] This supports our interpretation of the $A_i$ value as a circuit contribution to the aromatic stabilization energy (ASE). It is then termed the $i$-th circuit resonance energy ($CRE_i$).[31,32]

In general, an aromatic molecule is thermodynamically stable, in the sense that an external magnetic field destabilizes it energetically.[22] Therefore, an aromatic $\pi$-system is diamagnetic and enjoys the lowest energy when there is no magnetic field. A magnetic field diminishes the degree of cyclic conjugation.[22] Circuit-current susceptibility, i.e., the tendency of a given circuit to escape from the magnetic field, is proportional to the CRE multiplied by the circuit area. Ring-current diamagnetism represents the tendency of a $\pi$-system to retain ASE at the level of individual circuits.[31,32]

**Magnetic Resonance Energy.** Topological resonance energy (TRE) represents a typical ASE.[14,15,26] As the $A_i$ value can be interpreted as $CRE_i$, the sum of $A_i$ values over all circuits can also be interpreted as an ASE for an entire $\pi$-system. This quantity was termed magnetic resonance energy (MRE), which means a TRE-like quantity derived from the magnetic response of the $\pi$-system:[31–34]

$$\begin{aligned} \mathrm{MRE}/|\beta| &= \sum_{i}^{\mathrm{G}} A_i \\ &= \sum_{i}^{\mathrm{G}} \mathrm{CRE}_i/|\beta| \end{aligned} \tag{5}$$

where $i$ runs over all circuits in G. As has been reported for polycyclic aromatic hydrocarbons and their molecular ions,[31–34] MRE highly correlates in magnitude with TRE, so can be used safely as a kind of ASE.

In fact, CRE and MRE are absolute measures of aromatic stabilization, in the sense of not requiring reference standards for their qualification. Note that hypothetical polyene references are necessary to evaluate Dewar and de Llano,[44] Hess and Schaad,[45] and topological resonance energies.[14,15,26] Thus, CRE and MRE are recommended as being realistic measures of aromatic stabilization.

## Results and Discussion

Free-base (**1**) and metallated (**2**) porphines are iso-$\pi$-electronic with each other with 26$\pi$-electrons. These molecules have four pyrrolic rings; those in **1** are distinguished from each other by assigning different symbols (A, B, C, and D) to them. We did not distinguish between metalloporphine and the porphine dianion. The geometries of **1** and **2** employed to calculate magnetic properties are those obtained by Jusélius and Sundholm[10,11] using the resolution-of-the-identity density-functional theory (RI-DFT)[46] with the Becke–Perdew (B–P) parametrization.[47–49] For comparison's sake, the isostructural, iso-$\pi$-electronic hydrocarbon dianion **3** is assumed to have the same geometry as **2**. Gimarc termed such a reference hydrocarbon a uniform reference frame.[50]

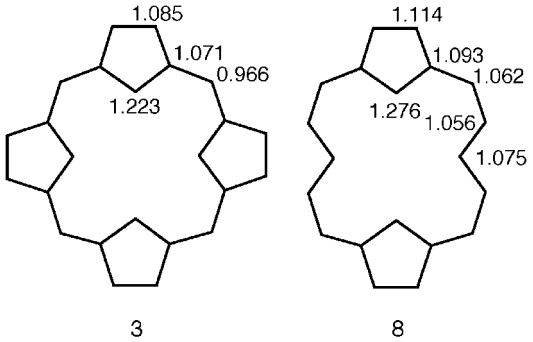


**Figure 3.** $\pi$-Electron densities of the uniform reference frames for porphines **3** and bacteriochlorin **8**.

The $\pi$-electron density map of the uniform reference frame **3** is shown in Figure 3. Longuet-Higgins et al. pointed out that the great stability of porphyrins is largely due to the presence of nitrogen atoms in the skeleton where, for purely geometrical reasons, the $\pi$-electrons tend to congregate.[51] Conversely, the best placement of electronegative nitrogen atoms is at the positions with the greatest charge in the uniform reference frame.[50,51] All nitrogen atoms in porphyrins **1** and **2** occupy the positions corresponding to those of highest charge density in **3**. Porphyrins thus conform to the rule of topological charge stabilization. We previously showed that almost all amine (>N–) nitrogens in biomolecules obey the rule of topological charge stabilization.[52]

TRE has been used as an energetic criterion of aromaticity for a wide variety of cyclic $\pi$-systems.[14,15,26] The TREs for porphyrins and related species are listed in Table 1. Both free-base porphine (**1**) and metalloporphine **2** have positive TREs comparable in magnitude to that of tricyclic anthracene ($\mathrm{TRE} = 0.475|\beta|$).[14,15,34] The percentage TRE (%TRE), an ASE normalized with respect to the size of a $\pi$-system, is defined as 100 times TRE, divided by the total $\pi$-binding energy of the polyene reference.[16,17,20,26] Those for **1** and anthracene are 1.42 and 2.52, respectively, indicating that the large $\pi$-systems of **1** and **2** are not highly but moderately aromatic. These porphyrins have slightly larger TREs than the uniform reference frame **3**. Stabilization of a $\pi$-system relative to the uniform reference frame is nothing other than a manifestation of topological charge stabilization in **1** and **2**.

Superaromatic stabilization energies (SSE) represents an extra stabilization energy due to macrocyclic conjugation.[16,17] It constitutes part of TRE. For porphyrins, SSE is equal to the bond resonance energy (BRE) for any of the CC bonds that link four pyrrolic rings. BRE is obtained by assigning a pair of imaginary resonance integrals ($\pm i\beta$) to a given $\pi$-bond.[17] SSE defined in this manner will be referred to as $t$-SSE, because another definition of SSE is introduced later. Here, letter "$t$" means a topologically defined quantity. One sees from Table 1 that $t$-SSE is not very large for **1** and **2**, amounting to only 17–20% of the total TRE. This amount of $t$-SSE definitely shows that macrocyclic conjugation never gives a major contribution to the global aromaticity of porphyrins

**Table 1.** TREs and *t*-SSEs for Porphyrins and Related Species

| Species | TRE/$\|\beta\|$ | *t*-SSE/$\|\beta\|$ | % *t*-SSE |
|---|---|---|---|
| Free-base porphine (**1**) | 0.4322 | 0.0843 | 20 |
| Metalloporphine **2** | 0.4744 | 0.0795 | 17 |
| Uniform reference frame for **1** and **2** (**3**) | 0.3941 | 0.0598 | 15 |
| Open-chain analogue **4** | 0.3937 | 0.0 | 0 |
| Open-chain analogue **5** | 0.3519 | 0.0 | 0 |
| Open-chain analogue **6** | 0.4122 | 0.0 | 0 |
| Bacteriochlorin (**7**) | 0.3171 | 0.0884 | 28 |
| Uniform reference frame for **7** (**8**) | 0.3143 | 0.0705 | 22 |
| Tetraaza[16]annulene dianion (**9**) | 0.0941 | — | — |

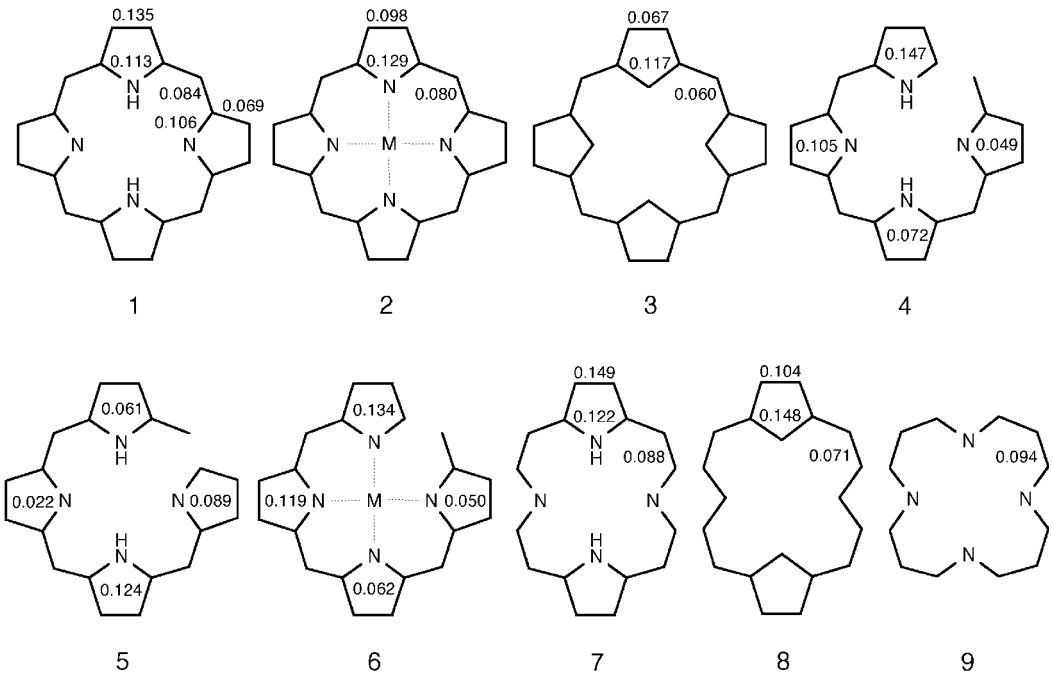


**Figure 4.** BREs for porphyrins and related species.

even though it is not negligibly small. It follows that neither the [18]annulene-like conjugation pathway in **1** nor the internal cross in **2** is crucial for determining the aromaticity of the entire $\pi$-system. The uniform reference frame **3** exhibits a bit smaller *t*-SSE. It, however, is true that the *t*-SSEs for **1**–**3** are much larger than those for kekulene and cyclacenes.[16,17] Cross-conjugation in these macrocycles must suppress superaromaticity to a great extent.

The BRE for any $\pi$-bond represents the contribution of all circuits that share the bond to TRE.[18–21] $\pi$-Bonds shared by the same set of rings necessarily have the same BRE. BREs for porphyrins and related species are graphically shown in Figure 4. Free-base and metallated porphyrins have no explicit antiaromatic substructures; all $\pi$-bonds have positive BREs, contributing more or less to the aromaticity of an entire $\pi$-system. As has been mentioned, the BREs for the CC bonds that link four pyrrolic rings are fairly small. $\pi$-Bonds in pyrrolic rings, in particular, rings A and C in **1** and all pyrrolic rings in **2**, exhibit larger BREs, suggesting that these rings are the main source of aromaticity. Such a situation is depicted as **1c** and **2c** in Figure 2. Main aromatic conjugation pathways are shown in bold in **1c** and **2c**.

Further supporting data for pyrrolic rings as a major source of aromaticity are obtained from the open-chain analogues **4**–**6**. These non-superaromatic molecules structurally resemble **1** or **2** but are devoid of one of the CC bonds needed to complete macrocyclic conjugation. Therefore, the BREs vanish for all of the CC bonds that link pyrrolic rings. The *t*-SSE necessarily vanishes. Nevertheless, these open-chain species still have as large positive TREs as **1** and **2**. The full retention of aromaticity in **4**–**6** is consistent with the small BREs for the CC bonds that link pyrrolic rings in **1** and **2** and again supports the view that pyrrolic rings contribute significantly to the aromaticity not only in porphyrins but also in such open-chain species as **4**–**6**.

We then examine aromaticity and ring-current diamagnetism in terms of individual circuits. As shown in Figure 5, there are eleven and six non-identical circuits in **1** and **2**, respectively, both species having 20 circuits in all. Four of them are located along the four pyrrolic rings. The rest of the circuits are located along the macrocycle that encloses the inner cavity. Each circuit in free-base porphine (**1**) is denoted by the number of sites (i.e., conjugated atoms) through which it passes and the nitrogen atoms involved. For example, $Z_5$(NH) and $Z_5$(N) in-

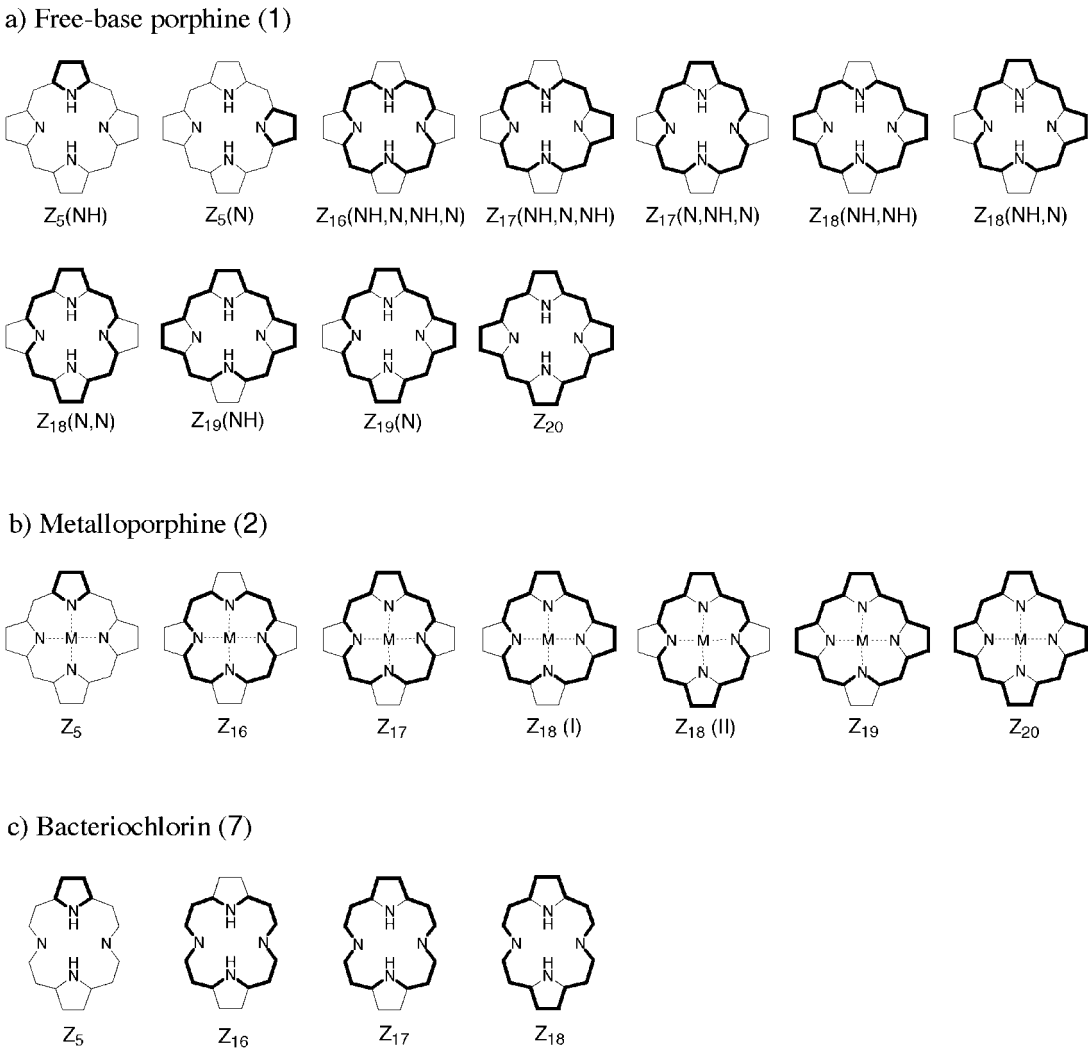


**Figure 5.** Non-identical circuits in free-base porphine (**1**), metalloporphine **2**, and bacteriochlorin (**7**).

dicate five-site circuits that pass through amine (>NH) and imine (=N–) nitrogens, respectively. A circuit located along the internal cross is denoted by $Z_{16}$(NH,N,NH,N), which indicates a 16-site circuit that passes through two amine and two imine nitrogens. Circuits in other species can be distinguished from each other simply by referring to the number of sites through which the circuit passes. For **2**, $Z_{18}$(I) is graph-theoretically identical with $Z_{18}$(II) although they are different in shape. Therefore, both are denoted simply by $Z_{18}$. A circuit taken along the internal cross in **2** is denoted by $Z_{16}$. All possible circuits in **3** are structurally identical with those in **2**.

The CREs calculated for all circuits in porphyrins and related species are listed in Table 2. It is noteworthy that all circuits in porphyrins **1** and **2** are aromatic with positive CREs; their common uniform reference frame **3** has slightly antiaromatic 19- and 20-site circuits. In general, smaller circuits have larger CREs.[32,33] All five-site circuits in **1** and **2** exhibit very large positive CREs, supporting the repeatedly mentioned view that the pyrrolic rings dominate the aromaticity of the entire molecule. $Z_{18}$(N,N) in **1**, which corresponds to an aromatic [18]annulene pathway, has the largest CRE among the circuits located along the macrocycle. However, it amounts to only one-sixth of the CRE for $Z_5$(NH). Likewise, the 16-site circuit in **2**, $Z_{16}$, which corresponds to the internal cross, has the largest CRE among the macrocyclic circuits. This CRE is again only one-sixth of the CRE for $Z_5$. For large circuits in **1** and **2**, cyclic conjugation energy (CCE) is comparable in magnitude to the CRE.[42] For **1** and **2**, high aromaticity of all five-site circuits is consistent with the locations of main conjugation pathways in **1c** and **2c**.

Magnetic resonance energy (MRE) is obtained by summing up all CREs,[31–34] whereas a kind of SSE is obtained as a sum of CREs for all circuits that enclose the inner cavity. This kind of SSE will be referred to as *m*-SSE, which means a magnetically defined quantity. MREs and *m*-SSEs for porphyrins and related species are listed in Table 3. Large porphyrin $\pi$-systems **1** and **2** exhibit positive MREs comparable in magnitude to that of tricyclic anthracene (MRE $= 0.341|\beta|$). They have slightly larger MREs than the uniform reference frame **3**. The *m*-SSE is not so large, amounting to 17–20% of the total MRE. Open-chain analogues exhibit MREs as large as porphyrins. Thus, MREs and *m*-SSEs again indicate that macrocyclic conjugation or a set of 16 large circuits in porphyrins never make a major contribution to the aromaticity. Note that essentially the same conclusion has been drawn from the TREs and *t*-SSEs. Such a parallelism between TRE and MRE supports

**Table 2.** Circuit Resonance Energies (CREs), Circuit Currents (CCs), and Circuit-Current Susceptibilities (CCSs) for Porphyrins and Related Species

| Circuit | Area/$S_0$ | Multiplicity | CRE/$\|\beta\|$ | CC/$I_0$ | CCS/$\chi_0$ |
|---|---|---|---|---|---|
| a) Free-base porphine (**1**) | | | | | |
| $Z_5$(NH) | 0.6671 | 2 | 0.0780 | 0.2342 | 0.1562 |
| $Z_5$(N) | 0.6688 | 2 | 0.0571 | 0.1720 | 0.1150 |
| $Z_{16}$(NH,N,NH,N) | 5.2476 | 1 | 0.0050 | 0.1179 | 0.6185 |
| $Z_{17}$(NH,N,NH) | 5.9164 | 2 | 0.0022 | 0.0596 | 0.3527 |
| $Z_{17}$(N,NH,N) | 5.9146 | 2 | 0.0082 | 0.2174 | 1.2858 |
| $Z_{18}$(NH,NH) | 6.5851 | 1 | 0.0009 | 0.0254 | 0.1671 |
| $Z_{18}$(NH,N) | 6.5834 | 4 | 0.0035 | 0.1028 | 0.6768 |
| $Z_{18}$(N,N) | 6.5817 | 1 | 0.0131 | 0.3890 | 2.5601 |
| $Z_{19}$(NH) | 7.2522 | 2 | 0.0013 | 0.0409 | 0.2964 |
| $Z_{19}$(N) | 7.2505 | 2 | 0.0053 | 0.1733 | 1.2563 |
| $Z_{20}$ | 7.9193 | 1 | 0.0018 | 0.0649 | 0.5142 |
| b) Metalloporphine **2** | | | | | |
| $Z_5$ | 0.6695 | 4 | 0.0767 | 0.2312 | 0.1548 |
| $Z_{16}$ | 5.2423 | 1 | 0.0129 | 0.3051 | 1.5996 |
| $Z_{17}$ | 5.9117 | 4 | 0.0066 | 0.1759 | 1.0400 |
| $Z_{18}$ | 6.5812 | 6 | 0.0034 | 0.1002 | 0.6597 |
| $Z_{19}$ | 7.2506 | 4 | 0.0011 | 0.0374 | 0.2714 |
| $Z_{20}$ | 7.9201 | 1 | 0.0006 | 0.0216 | 0.1710 |
| c) Uniform reference frame for **1** and **2** (**3**) | | | | | |
| $Z_5$ | 0.6695 | 4 | 0.0684 | 0.2061 | 0.1380 |
| $Z_{16}$ | 5.2423 | 1 | 0.0283 | 0.6677 | 3.5005 |
| $Z_{17}$ | 5.9117 | 4 | 0.0051 | 0.1354 | 0.8005 |
| $Z_{18}$ | 6.5812 | 6 | 0.0002 | 0.0062 | 0.0411 |
| $Z_{19}$ | 7.2506 | 4 | −0.0002 | −0.0063 | −0.0456 |
| $Z_{20}$ | 7.9201 | 1 | −0.0000 | −0.0010 | −0.0078 |
| d) Bacteriochlorin (**7**) | | | | | |
| $Z_5$ | 0.6671 | 2 | 0.0882 | 0.2647 | 0.1766 |
| $Z_{16}$ | 5.2476 | 1 | 0.0097 | 0.2282 | 1.1972 |
| $Z_{17}$ | 5.9146 | 2 | 0.0168 | 0.4438 | 2.6520 |
| $Z_{18}$ | 6.5817 | 1 | 0.0286 | 0.8478 | 5.5797 |
| e) Uniform reference frame for **7** (**8**) | | | | | |
| $Z_5$ | 0.6671 | 2 | 0.0994 | 0.2985 | 0.1991 |
| $Z_{16}$ | 5.2476 | 1 | 0.0378 | 0.8923 | 4.6822 |
| $Z_{17}$ | 5.9146 | 2 | 0.0096 | 0.2545 | 1.5050 |
| $Z_{18}$ | 6.5817 | 1 | 0.0010 | 0.0298 | 0.1962 |
| f) Tetraaza[16]annulene dianion (**9**) | | | | | |
| $Z_{16}$ | 5.2423 | 1 | 0.0765 | 1.8042 | 9.4581 |

the utility and reasonableness of the MRE concept.

Circuit currents (CCs) and circuit-current susceptibilities (CCSs) for porphyrins and related species, each expressed as a multiple of the benzene value, are added in Table 2. Ring current maps obtained by superposing all these circuit currents are presented in Figure 6. Here, diamagnetic circulation is shown as counterclockwise, paramagnetic as clockwise. These $\pi$-current density maps are exactly the same as those obtained by the conventional Hückel–London method.[28–30,52–54] Steiner and Fowler[12,13] carried out more elaborate ring-current calculations with the 6-31G** basis by means of coupled Hartree–Fock theory within the ipsocentric[55,56] CTOCD-DZ (continuous transformation of origin of current density diamagnetic zero) formulation.[57,58] The $\pi$-current distributions they obtained for free-base porphine (**1**) and magnesium porphine[12] are qualitatively very similar to ours for **1** and **2**, respectively. Thus, we can safely say that the present approach provides reasonable physical pictures for porphyrins.

Porphyrins possess a macrocyclic ring current in appearance. A main path of circulation around the porphyrin macro-

**Table 3.** MREs and *m*-SSEs for Porphyrins and Related Species

| Species | MRE/$\|\beta\|$ | *m*-SSE/$\|\beta\|$ | % *m*-SSE |
|---|---|---|---|
| Free-base porphine (**1**) | 0.3390 | 0.0686 | 20 |
| Metalloporphine **2** | 0.3718 | 0.0649 | 17 |
| Uniform reference frame for **1** and **2** (**3**) | 0.3227 | 0.0491 | 15 |
| Open-chain analogue **4** | 0.3678 | 0.0 | 0 |
| Open-chain analogue **5** | 0.3360 | 0.0 | 0 |
| Open-chain analogue **6** | 0.3820 | 0.0 | 0 |
| Bacteriochlorin (**7**) | 0.2484 | 0.0720 | 29 |
| Uniform reference frame for **7** (**8**) | 0.2568 | 0.0579 | 23 |
| Tetraaza[16]annulene dianion (**9**) | 0.0765 | — | — |

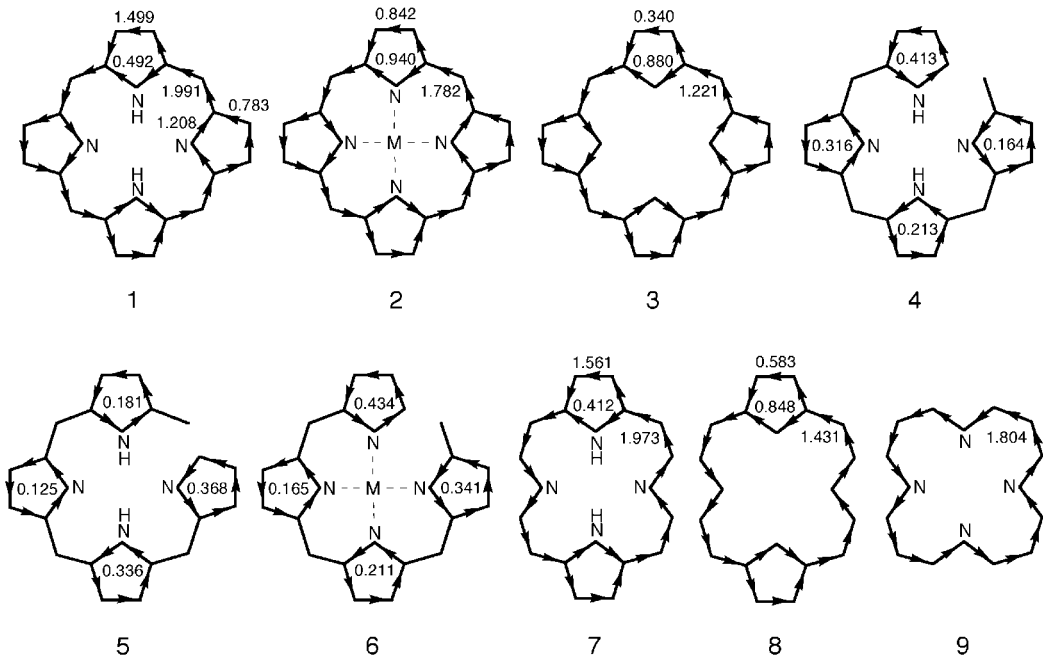


**Figure 6.** $\pi$-Current density maps for porphyrins and related species. All current intensities are expressed as a ratio to the benzene value.

cycle depends strongly on occupation of the central region.[8,12,13] Removal of the metal ion would result in a two-way transfer of charge, whereby the $\sigma$ lone pair on the nitrogen moves away from the center, and the $\pi$-charge is drawn inward from the periphery.[13] The resulting circulation in the porphyrin dianion would closely follow the internal cross.[12,13] Our $\pi$-current distribution in **2** is similar to that for magnesium porphine,[12,13] because substantial bifurcation of the ring current is predicted to occur across every pyrrolic ring. Fortunately or not, polarization of the $\sigma$-framework cannot be taken into account in our simple model. It must be for this reason that our $\pi$-current density map for **2** happened to be very similar to that for magnesium porphine.

All of the three research groups agreed that not only macrocyclic conjugation but also pyrrolic rings are the main sources of aromaticity in porphyrins although the details of their discussions are not the same.[8–13] They depicted such a conjugation scheme on the basis of the global circulation of $\pi$-electrons and large negative NICS values at the centers of pyrrolic rings. Diamagnetic circulation in fact is supposed to span all or almost all conjugated atoms. Jusélius and Sundholm considered the total aromatic pathway of porphyrins as a superposition of several $(4n + 2)\pi$-electron pathways.[11] This way of reasoning is more or less similar to our graph-theoretical approach to ring and circuit currents. According to Steiner and Fowler,[12,13] a notable feature of the induced circulation in porphyrins is the bifurcation of the current across the pyrrole subunits.

We, however, should be very cautious when interpreting the NICS values calculated for individual rings in a polycyclic $\pi$-system.[59–62] If the $\pi$-current density patterns of **1** and **2** indeed are dominated by the global circulation of $\pi$-electrons, the NICS value at the center of each pyrrolic ring will reflect the bifurcation of the ring current. In fact, the inside of each pyrrolic ring is surrounded not only by three diatropic bonds (outer CC bonds) but also by two apparently paratropic bonds (inner CN bonds). Therefore, when it comes to a polycyclic $\pi$-system, the conventional interpretation of NICS values in terms of local aromaticity might give a misleading picture of aromaticity. In general, the intensity of a ring current induced in a polycyclic $\pi$-system does not always reflect the degree of aromaticity correctly.[59–62]

Equation 4 states that the intensity of the *i*-th circuit current is proportional not only to the $A_i$ value but also to the circuit

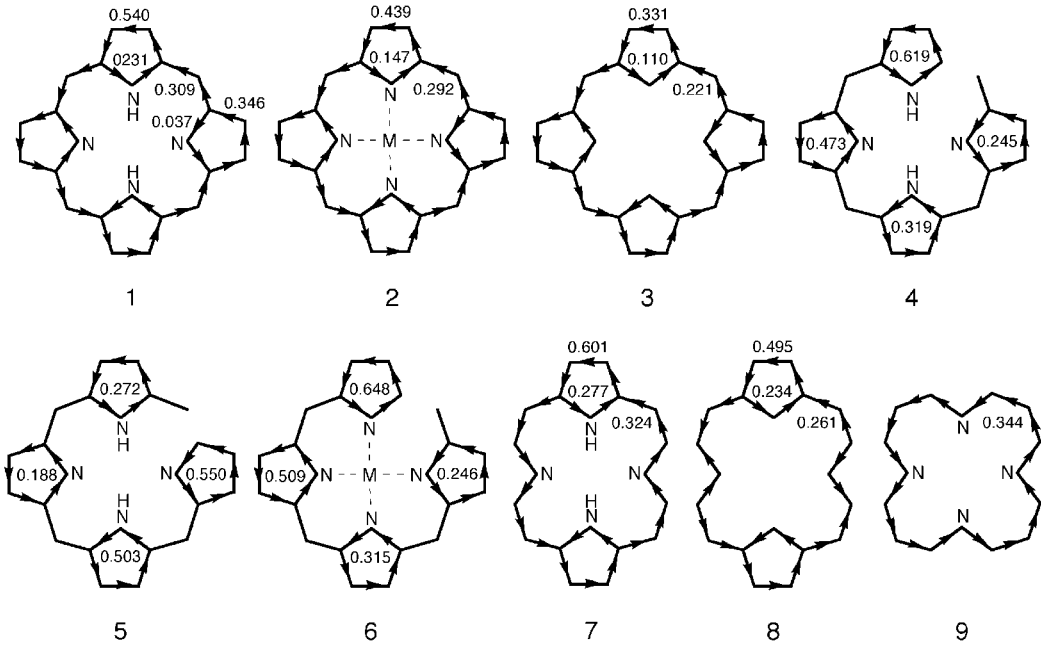


**Figure 7.** Maps of geometry-independent $\pi$-current density for porphyrins and related species. All current intensities are expressed as a ratio to the benzene value.

area $S_i$. As to free-base porphine (**1**), the largest circuit current is induced in $Z_{18}$(N,N), a circuit located along the 18$\pi$-[18]-annulene ring. Note, however, that the area of this circuit is 9.9 times as large as pyrrolic rings A and C and must be largely responsible for a strong $\pi$-current induced in it. This circuit current must be associated with the aromatic conjugation pathways **1a** and **1b** so far proposed for **1**. In the case of metalloporphine **2**, the largest current is induced in the large circuit located along the internal cross ($Z_{16}$). This is apparently consistent with aromatic pathways **2a** and **2b**. However, it goes without saying that large circuits, such as $Z_{18}$(N,N) in **1** and $Z_{16}$ in **2**, are far from being main sources of aromaticity even though they sustain large diamagnetic $\pi$-currents. Like a large [18]annulene $\pi$-system,[63] the high diatropicity of a large circuit does not indicate the high aromaticity of the circuit.

Steiner and Fowler attempted to predict the occurrence of net local circulation in individual rings of a polycyclic $\pi$-system.[64] We here apply their way of predicting local currents to free-base porphine (**1**). The bifurcated $\pi$-current density in rings A and C is much stronger on the periphery than along the inner route so that, viewed from the center of A, the net local circulation is counterclockwise, implying diamagnetism. The converse is true for rings B and D, with net local clockwise circulation, implying paramagnetism. All pyrrolic rings in **2** resemble rings B and D in **1**, so must sustain local paramagnetic currents. The present study revealed that this way of reasoning is not always applicable to polycyclic $\pi$-systems. All circuits in **1** and **2** were found to be diatropic, sustaining diamagnetic circuit currents. Both apparent diamagnetic and paramagnetic currents in pyrrolic rings arise from the superposition of many diamagnetic circuit currents.

As has been seen, both ring and circuit currents are dependent on molecular geometry. By applying the procedure proposed in Ref. 33, we can estimate the geometry-independent current densities for **1**–**6**. If a molecule is artificially deformed to reduce the areas of all rings or all circuits, the entire $\pi$-electron current will be more or less decreased. In order to escape from such a geometric dependence of $\pi$-electron currents, hypothetical geometry-independent $\pi$-electron currents were evaluated by equating all circuit areas in **1**–**6** with that of the benzene ring (i.e., $S_i = S_0$ for all circuits). Of course, it is impossible to deform a polycyclic $\pi$-system in this manner because all circuit areas are interdependent. The virtual or geometry-independent $\pi$-current density maps thus obtained for porphyrins and related species are shown in Figure 7. We observe in this figure that macrocyclic $\pi$-currents in **1**–**3** are greatly reduced in their respective maps of geometry-independent $\pi$-current density.

The intensity of a geometry-independent $\pi$-electron current running through a peripheral bond is roughly proportional to the corresponding BRE. If the current intensity is divided by 4.5, the approximate BRE value will be obtained. As suggested previously,[33] the geometry-independent current intensity for a peripheral $\pi$-bond represents well the degree of local aromaticity for the ring to which the bond belongs. Figure 7 shows that strong geometry-independent currents are supposed to run through the pyrrolic CC bonds. Consequently, local counterclockwise geometry-independent currents are clearly visible in individual pyrrolic rings in **1** and **2**. Such distributions of geometry-independent currents also indicate that the five-site circuits are the main source of aromaticity.

On the other hand, eq 2 indicates that the circuit-current susceptibility (CCS) is proportional to the CRE, weighted with the circuit area squared. Therefore, for large polycyclic $\pi$-systems, some large circuits with small CREs may dominate the ring-current susceptibility. For example, large circuits, such as $Z_{17}$(N,NH,N), $Z_{18}$(N,N), and $Z_{19}$(N) in **1** and $Z_{16}$ and $Z_{17}$ in **2**, contribute much more to the ring-current susceptibility. The most aromatic circuits, $Z_5$(NH) and $Z_5$(N) in **1** and $Z_5$ in **2**, make the least contribution to their respective ring-cur-

rent susceptibilities, because they have very small areas. In the case of free-base porphine (**1**), 80% of the MRE arises from the four five-site circuits, but these circuits make a contribution to only 4% of the ring-current susceptibility; this molecule has 13.5 times as large a ring-current susceptibility as benzene, whereas its TRE is only 1.6 times as large. Thus, the ring-current susceptibility or diamagnetic susceptibility exaltation cannot always be used as a reliable indicator of aromaticity, at least for polycyclic $\pi$-systems.

A comparison of **1** and **2** with the bacteriochlorin $\pi$-system **7** and the tetraaza[16]annulene dianion (**9**), shown in Figure 1, may be instructive, because the $\pi$-systems of these molecules contain the main aromatic conjugation pathways proposed for **1** and **2**.[8–13] Bacteriochlorin is a derivative of porphine, in which all $\beta$-positions of rings B and D are saturated with hydrogens. Doubly charged hydrocarbon **8** represents a uniform reference frame for **7**. The TREs and MREs for **7**–**9** are added in Tables 1 and 3, respectively. Bacteriochlorin **7** is very close in aromaticity to its uniform reference frame **8**. The BREs and the $\pi$-current densities for **7**–**9** are graphically shown in Figures 4 and 6, respectively. Geometries of these species were assumed to be the same as the corresponding parts of **1**–**3**. The $\pi$-current density maps for **7** and **9** indeed bear a close resemblance to those for **1** and **2**, respectively. However, they are much less aromatic with smaller TREs than the corresponding porphyrins, because they have two less or no pyrrolic rings. A comparison of actual $\pi$-current densities in Figure 6 with geometry-independent ones in Figure 7 again confirms that the main source of ring current is different from that of aromaticity. As may be seen from Figure 3, imine nitrogens do not always obey the rule of topological charge stabilization,[52] because it is only one $\pi$-electron that they provide for the entire $\pi$-system.

## Concluding Remarks

Ring currents are strongly dependent on molecular geometry, so that information on main aromatic conjugation pathways cannot be extracted straightforwardly from the $\pi$-current density. We interpreted the $\pi$-current density map and aromatic stabilization energy consistently within a single theoretical framework and found that main pathways of circulation along the macrocycle are not the same as those of aromatic stabilization. As pointed out properly by Heine et al.,[65] the term "aromatic" describes molecules that benefit energetically from the delocalization of mobile electrons in closed circuits. In this sense, the main source of aromaticity in porphyrins is four five-site circuits. As stated in previous papers,[59–62] energetic and magnetic criteria of aromaticity often make different predictions on the aromaticity of polycyclic $\pi$-systems.

Steiner and Fowler pointed out that the distinction between bifurcated flow and real local circulations is difficult when only global integrated magnetic properties are calculated.[64] In fact, no one has proposed bifurcation mechanisms in a polycyclic $\pi$-system or evidence of local ring currents in individual rings. We have seen that this problem can be solved readily using our graph theory. $\pi$-Currents are induced separately in all possible circuits in the porphine $\pi$-system. Superposition of all these circuit currents gives rise to the apparent bifurcation of the current across each pyrrolic unit. Local currents induced in pyrrolic rings are those induced in the corresponding five-site circuits.

Finally, we briefly refer to the kinetic stability of porphyrins. Porphyrinoids with $4n\pi$-electron conjugation pathways have been little explored owing to their synthetic difficulty.[6,66,67] According to our preliminary study, they are still aromatic with positive TREs, although $\pi$-bonds linking two adjacent five-membered rings often exhibit small negative BREs. Therefore, it seems quite likely that such an antiaromatic macrocycle is the source of kinetic instability or synthetic difficulty.[18–20] In general, kinetic instability is very sensitive to the presence or absence of anti- or non-aromatic substructures. From such a kinetic point of view, free-base porphine and metalloporphine may still be considered as [18]annulene derivatives, in which the CC bonds with small BREs link pyrrolic rings. Thus, macrocyclic conjugation cannot be disregarded in porphyrin chemistry when the kinetic stability is considered.

This work was supported by a Grant-in-Aid for Scientific Research (No. 16550016) from the Japan Society for the Promotion of Science. Computations were carried out at the Information Processing Center, Shizuoka University, and the Research Center for Computational Science, Okazaki National Research Institutes.

## References

1 *The Porphyrins*, ed. by D. Dolphin, Academic Press, New York, **1978–1979**, Vols. 1–8.
2 *The Porphyrin Handbook*, ed. by K. M. Kadish, K. M. Smith, R. Guilard, Academic Press, San Diego, **2000–2003**, Vols. 1–20.
3 E. B. Fleischer, *Acc. Chem. Res.* **1970**, *3*, 105.
4 D. Spangler, G. M. Maggiora, L. L. Shipman, R. E. Christoffersen, *J. Am. Chem. Soc.* **1977**, *99*, 7470.
5 D. Spangler, G. M. Maggiora, L. L. Shipman, R. E. Christoffersen, *J. Am. Chem. Soc.* **1977**, *99*, 7478.
6 E. Vogel, *Pure Appl. Chem.* **1993**, *65*, 143.
7 E. Vogel, *J. Heterocycl. Chem.* **1996**, *33*, 1461.
8 M. K. Cyrański, T. M. Krygowski, M. Wisiorowski, N. J. R. van Eikema Hommes, P. v. R. Schleyer, *Angew. Chem., Int. Ed.* **1998**, *37*, 177.
9 T. M. Krygowski, M. K. Cyrański, *Chem. Rev.* **2001**, *101*, 1385.
10 J. Jusélius, D. Sundholm, *Phys. Chem. Chem. Phys.* **2000**, *2*, 2145.
11 J. Jusélius, D. Sundholm, *J. Org. Chem.* **2000**, *65*, 5233.
12 E. Steiner, P. W. Fowler, *ChemPhysChem* **2002**, *3*, 114.
13 E. Steiner, A. Soncini, P. W. Fowler, *Org. Biomol. Chem.* **2005**, *3*, 4053.
14 J. Aihara, *J. Am. Chem. Soc.* **1976**, *98*, 2750.
15 I. Gutman, M. Milun, N. Trinajstić, *J. Am. Chem. Soc.* **1977**, *99*, 1692.
16 J. Aihara, *J. Am. Chem. Soc.* **1992**, *114*, 865.
17 J. Aihara, *J. Chem. Soc., Faraday Trans.* **1995**, *91*, 237.
18 J. Aihara, *J. Am. Chem. Soc.* **1995**, *117*, 4130.
19 J. Aihara, *J. Phys. Chem.* **1995**, *99*, 12739.
20 J. Aihara, *J. Chem. Soc., Perkin Trans. 2* **1996**, 2185.
21 J. Aihara, T. Ishida, H. Kanno, *Bull. Chem. Soc. Jpn.* **2007**, *80*, 1518.

22 J. Aihara, *J. Am. Chem. Soc.* **1979**, *101*, 558.
23 J. Aihara, *J. Am. Chem. Soc.* **1979**, *101*, 5913.
24 J. Aihara, *Bull. Chem. Soc. Jpn.* **1981**, *54*, 1245.
25 J. Aihara, *J. Am. Chem. Soc.* **1981**, *103*, 5704.
26 J. Aihara, *Pure Appl. Chem.* **1982**, *54*, 1115.
27 J. Aihara, T. Horikawa, *Chem. Phys. Lett.* **1983**, *95*, 561.
28 J. Aihara, *Bull. Chem. Soc. Jpn.* **1983**, *56*, 1853.
29 J. Aihara, *J. Am. Chem. Soc.* **1985**, *107*, 298.
30 J. Aihara, *Bull. Chem. Soc. Jpn.* **1985**, *58*, 1045.
31 J. Aihara, *Bull. Chem. Soc. Jpn.* **2004**, *77*, 651.
32 J. Aihara, *J. Am. Chem. Soc.* **2006**, *128*, 2873.
33 J. Aihara, H. Kanno, *J. Phys. Chem. A* **2005**, *109*, 3717.
34 J. Aihara, H. Kanno, T. Ishida, *J. Phys. Chem. A* **2007**, *111*, 8873.
35 F. London, *J. Phys. Radium* **1937**, *8*, 397.
36 B. Pullman, A. Pullman, *Les Théories Electroniques de la Chimie Organique*, Masson et Cie, Paris, **1952**, Chap. IX.
37 F. A. Van-Catledge, *J. Org. Chem.* **1980**, *45*, 4801.
38 A. Graovac, I. Gutman, N. Trinajstić, T. Zivković, *Theor. Chim. Acta* **1972**, *26*, 67.
39 J. A. Pople, K. G. Untch, *J. Am. Chem. Soc.* **1966**, *88*, 4811.
40 S. Bosanac, I. Gutman, *Z. Naturforsch., A: Phys. Sci.* **1977**, *32*, 10.
41 I. Gutman, S. Bosanac, *Tetrahedron* **1977**, *33*, 1809.
42 I. Gutman, *Croat. Chem. Acta* **1985**, *58*, 359.
43 I. Gutman, *Monatsh. Chem.* **2005**, *136*, 1055.
44 M. J. S. Dewar, C. de Llano, *J. Am. Chem. Soc.* **1969**, *91*, 789.
45 B. A. Hess, Jr., L. J. Schaad, *J. Am. Chem. Soc.* **1971**, *93*, 305.
46 K. Eichkorn, O. Treutler, H. Öhm, M. Häser, R. Ahlrichs, *Chem. Phys. Lett.* **1995**, *240*, 283.
47 S. H. Vosko, L. Wilk, M. Nusair, *Can. J. Phys.* **1980**, *58*, 1200.
48 J. P. Perdew, *Phys. Rev. B* **1986**, *33*, 8822.
49 A. D. Becke, *Phys. Rev. A* **1988**, *38*, 3098.
50 B. M. Gimarc, *J. Am. Chem. Soc.* **1983**, *105*, 1979.
51 H. C. Longuet-Higgins, C. W. Rector, J. R. Platt, *J. Chem. Phys.* **1950**, *18*, 1174.
52 J. Aihara, *Bull. Chem. Soc. Jpn.* **1988**, *61*, 3129.
53 J. A. Pople, *Mol. Phys.* **1958**, *1*, 175.
54 R. McWeeny, *Mol. Phys.* **1958**, *1*, 311.
55 E. Steiner, P. W. Fowler, *J. Phys. Chem. A* **2001**, *105*, 9553.
56 E. Steiner, P. W. Fowler, *Chem. Commun.* **2001**, 2220.
57 T. A. Keith, R. F. W. Bader, *Chem. Phys. Lett.* **1993**, *210*, 223.
58 S. Coriani, P. Lazzeretti, M. Malagoli, R. Zanasi, *Theor. Chim. Acta* **1994**, *89*, 181.
59 J. Aihara, *Chem. Phys. Lett.* **2002**, *365*, 34.
60 J. Aihara, S. Oe, *Bull. Chem. Soc. Jpn.* **2003**, *76*, 1363.
61 J. Aihara, *Chem. Phys. Lett.* **2004**, *393*, 7.
62 J. Aihara, *J. Phys. Org. Chem.* **2005**, *18*, 235.
63 F. Sondheimer, *Acc. Chem. Res.* **1972**, *5*, 81.
64 E. Steiner, P. W. Fowler, *Org. Biomol. Chem.* **2003**, *1*, 1785.
65 T. Heine, P. v. R. Schleyer, C. Corminboeuf, G. Seifert, R. Reviakine, J. Weber, *J. Phys. Chem. A* **2003**, *107*, 6470.
66 J. L. Sessler, D. Seidel, *Angew. Chem., Int. Ed.* **2003**, *42*, 5134.
67 C. Liu, D.-M. Shen, Q.-Y. Chen, *J. Am. Chem. Soc.* **2007**, *129*, 5814, and references cited therein.